Electron correlations and fractional quantum Hall states in a double-layer electron system

LETTER TO THE EDITOR

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Abstract. An analytically solvable few-electron model of a double-layer electron system is presented which includes mixing with all Landau levels. Inter-layer correlations are found to give rise to novel ground states which differ fundamentally from previously suggested Jastrow-like wavefunctions for double layer N-electron systems. Comparison with experimental data suggests that these states may represent small-N precursors of recently observed fractional quantum Hall states in double-layer (large-N) systems.

An exciting recent development in the field of low-dimensional electron gases is the experimental discovery in double-layer electron systems of novel fractional quantum Hall effect (FQHE) states at $\frac{1}{2}$ and $\frac{1}{4}$ filling per layer [1, 2]. For a single layer, the intra-layer correlations giving rise to ground states at observed filling fractions $1/m$ (m odd) can be modelled using Jastrow wavefunctions [3]. In a double-layer system, the ground state will also depend on inter-layer correlations [4, 5]. Due to the intractability of the N-electron problem, all previous cluster (i.e. small-N) calculations on the double-layer system have been numerical and computationally intensive; results have been reported for as few as $N = 6$ electrons (three electrons per layer). Such studies investigate the form of the resulting N-electron wavefunction by means of numerical overlap with known (usually Jastrow-like or so-called Greek–Roman) analytic forms and are usually limited to the lowest Landau level. This paper seeks to investigate *directly* the analytic form of the double-layer correlated N-electron wavefunction by means of an analytically solvable few-electron model which includes mixing with all higher Landau levels. We find that inter-layer interactions give rise to a series of novel correlated ground states. The analytically obtained wavefunctions for these ground states are fundamentally distinct from previously proposed Jastrow forms. As with all cluster studies, the results of our small-N model cannot be guaranteed to carry over to the large-N limit; however a comparison with experimental data on the double-layer FQHE [1] suggests that these novel ground states may indeed represent small-N precursors of the recently observed fractional states.

As in various other cluster studies (e.g. [6]) we take the electrons as moving within each layer (xy plane) in a two-dimensional parabolic potential $\hbar\omega_0$, subject to a magnetic field B along the z direction. The two layers labelled by $\alpha = 1, 2$ have separation s along z. We neglect inter-layer tunnelling of the electrons and assume strong z direction confinement within each layer (electrons frozen in lowest z subband). These are reasonable starting approximations for the AT&T double-layer samples [1]. We consider the effect of just two electrons per layer. The resulting four-electron model, which will be solved analytically, contains just one electron less per layer than several recent numerical calculations [4];

it is the smallest-N model exhibiting both inter-layer and intra-layer electron–electron interactions. With a symmetric gauge, the four-electron Hamiltonian is $H = H_0 + V$ with

$$H_0 = \sum_{i,\alpha} \left(\frac{p_{i,\alpha}^2}{2m^*} + \frac{1}{2} m^* \omega_0^2(B) |r_{i,\alpha}|^2 + \frac{\omega_c}{2} L_{i,\alpha} \right) \tag{1}$$

where $p_{i,\alpha}$ and $r_{i,\alpha}$ are the electron momentum and position in the xy plane with $i = 1,2$. Each electron has effective mass m^*, z component of angular momentum $L_{i,\alpha}$, and becomes spin-polarized for the B fields of interest. The cyclotron frequency is ω_c and $\omega_0^2(B) = \omega_0^2 + \omega_c^2/4$. We take the electron–electron interaction potential to be of inverse-square form

$$V = \sum_\alpha \frac{\beta}{|r_{1,\alpha} - r_{2,\alpha}|^2} + \sum_{i,j} \frac{\beta}{|r_{i,1} - r_{j,2}|^2 + s^2} \tag{2}$$

where β is a positive parameter. The inverse-square interaction for a single layer gives quantitatively similar results to the bare Coulomb interaction [7]. We introduce coordinates $u_1 = \frac{1}{2} \sum_{i,\alpha} r_{i,\alpha}$ (centre-of-mass), $u_2 = 2^{-1/2}(r_{1,1} - r_{2,1})$ (relative coordinate in layer 1), $u_3 = 2^{-1/2}(r_{1,2} - r_{2,2})$ (relative coordinate in layer 2), $u_4 = \frac{1}{2}\{(r_{1,1} + r_{2,1}) - (r_{1,2} + r_{2,2})\}$ (relative coordinate between centres-of-mass for layers 1 and 2) together with the corresponding momenta q_i. This transformation leaves H_0 unchanged. We now make the assumption that the average separation between electrons in the xy plane is less than the layer separation s. The second (i.e. inter-layer) term in (2) can hence be expanded as an infinite Taylor series in $(r/s)^2$, where $r \equiv |r_{i,\alpha} - r_{j,\alpha'}|$, with nth term $\beta r^{2n-2}/s^{2n}$.

We first consider a large enough layer separation s such that this Taylor series can be truncated at its second term, hence becoming $\beta/s^2 - \beta r^2/s^4$. The Hamiltonian H becomes

$$H = \frac{4\beta}{s^2} + \sum_{i=1}^4 H_i(u_i, q_i). \tag{3}$$

H_1 and H_4 are Hamiltonians for a single electron in a parabolic potential ω_i and magnetic field B; H_2 and H_3 are Hamiltonians for an electron pair moving in a parabolic potential ω_i and magnetic field B with inverse-square interaction of strength $\beta/2$. The potentials $\omega_1 = \omega_0$, $\omega_2 = (\omega_0^2 - 4\beta/m^*s^4)^{1/2} = \omega_3$, and $\omega_4 = (\omega_0^2 - 8\beta/m^*s^4)^{1/2}$. Each H_i (and hence H) is exactly solvable analytically. The total energy E is $\sum_{i=1}^4 E_i$. Here E_1 and E_4 are well-known one-electron spectra with angular momenta m_1 and m_4. For $i = 2, 3$

$$E_i = \hbar \omega_i(B)(2n_i + ((\beta m^*/\hbar^2) + m_i^2)^{1/2} + 1) - m_i \hbar \omega_c/2 \tag{4}$$

where n_i and m_i are integers[7]; here $\omega_i^2(B) = \omega_i^2 + \omega_c^2/4$. The lowest energy states E have $n_i = 0 = m_1 = m_4$, but may have relative angular momenta m_2 and m_3 greater than zero since E_2 and E_3 have minima at *finite* $m \sim (\beta m^*/\hbar^2)^{1/2}(\omega_c/2\omega_2)$. The total ground state angular momentum $J = m_2 + m_3$; m_2 and m_3 must be odd to satisfy the antisymmetry requirement. (Zero inter-layer tunnelling implies that electrons in different layers are effectively distinguishable). The sequence of ground states $|m_2, m_3\rangle$ with increasing B field is $|1, 1\rangle$, $|3, 3\rangle$, $|5, 5\rangle$, ..., $|n, n\rangle$. We will now attempt to identify an effective filling factor for these states in our small-N system. A recent study has concluded that an appropriate definition of an effective filling factor ν in small-N systems is $\nu = (N - 1)/m_{\text{max}}$ where m_{max} is the *maximum possible single-electron angular momentum* [8]. Hence for each layer $\nu = 1/m_{\text{max}}$ [8]; the total effective filling factor of the two-layer system is therefore $\nu = \sum_\alpha \nu_\alpha$ where $\nu_1 = 1/m_2$ and $\nu_2 = 1/m_3$. The resulting ground state sequence for total ν is 2, 2/3, 2/5 which is actually consistent with observed fractions for two distant planes (e.g. sample D in [1]).

For smaller values of the layer separation s we must include the next term in the inter-layer Taylor expansion, given by $\beta f(\{u_i\})/s^6$ where $f(\{u_i\})$ is a sum of six terms of the form $(u_i \cdot u_i)(u_j \cdot u_j)$ and three of the form $(u_i \cdot u_j)^2$. All these produce energy perturbations, but only the latter give rise to significant wavefunction mixing (NB: V conserves J). In particular, $(u_2 \cdot u_3)^2$ mixes the previously degenerate states $|3, 1\rangle$ and $|1, 3\rangle$ (also $|3, 5\rangle$ and $|5, 3\rangle$, ... $|n, n+2\rangle$ and $|n+2, n\rangle$). This mixing yields two *entangled* states $|3, 1\rangle_+ \sim (|3, 1\rangle + |1, 3\rangle)$ and $|3, 1\rangle_- \sim (|3, 1\rangle - |1, 3\rangle)$, and similarly for $|n+2, n\rangle_\pm$. The energies of $|n+2, n\rangle_\pm$ are found analytically using degenerate perturbation theory [9]. For any n, a range of B fields exist where $|n+2, n\rangle_-$ lies below both $|n, n\rangle$ and $|n+2, n+2\rangle$. To obtain the total ν for these states, we employ $\nu_\alpha = 1/\bar{m}_\alpha$ where $\bar{m}_\alpha$ is the average m in layer α. For $|3, 1\rangle_-$, $\bar{m}_\alpha = \frac{1}{2}(3) + \frac{1}{2}(1) = 2$ and the total $\nu = \frac{1}{2} + \frac{1}{2} = 1$. For $|5, 3\rangle_-$, $\nu = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

Figure 1(a) shows the energy curves for the eigenstates $|n, n\rangle$ and $|n+2, n\rangle_-$ as a function of B for our small-N model. Figure 1(b) gives the experimental results for the (large-N) double-layer electron system (adapted from figure 1 of [1]). Our model parameters are chosen to be consistent with the experiment. The sample electron density is 10^{11} cm^{-2}. We only have two electrons per layer but can approximate such a density by choosing the xy parabolic potential $\hbar\omega_0 = 0.8$ meV; this gives an average density across the electron distribution of $\sim 10^{11}$ cm^{-2}. The full width of the double-layer structure is 391 Å [1]. Choosing $s = 391$ Å partially compensates for wavefunction spread along the z direction in the experiment; it is also consistent with our earlier assumption that the average xy electron–electron separation be less than s. The parameter β is chosen so that the inverse-square interaction coincides with the Coulomb interaction at $r \sim 30$ Å. It turns out that the *qualitative features of figure 1(a) are fairly insensitive to changes in β and $\hbar\omega_0$.*

Considering the simplicity of our analytic model and the obvious limitations of such a small-N analysis, there is a surprising correspondence between the positions of the observed FQHE states and the occurrence of the theoretical ground states. For example, figure 1(a) suggests the $\nu = \frac{1}{2}$ state will occur near $B = 9$ T (i.e. B value of the largest $|5, 3\rangle_-$ excitation gap) and disappear for $T \sim 0.5$ K. This corresponds well to figure 1(b). If B is increased beyond 10 T (not shown) the $|n+2, n\rangle_-$ states eventually exclude the $|n, n\rangle$ states from becoming ground states. The theory therefore suggests that certain small ν fractions (e.g. $\frac{2}{7}$) which occur for two distant layers may be *absent* for intermediate s. The inset in figure 1(a) shows how the regions of stability of the theoretical ground states vary as s increases. As found experimentally the $\nu = 1$ and $\nu = \frac{1}{2}$ states rapidly lose their stability, leaving just the sequence $\nu = 2, \frac{2}{3}, \frac{2}{5}, \ldots$ for large s.

Figure 2 compares the charge densities for the analytically obtained eigenstates $|3, 3\rangle$ and $|3, 1\rangle_-$. Each plot gives the charge density for one layer, say layer 1, with the electrons in layer 2 fixed in the classical configuration for a single layer (diametrically opposite each other on the x axis at $y = 0$). For $|3, 3\rangle$, the electrons in layer 1 show no visible correlation with those in layer 2. For $|3, 1\rangle_-$, the electrons in layer 1 become localized along $x = 0$ in an effort to minimize the inter-layer electrostatic energy; the four electrons are approaching the classical 'crystal-like' configuration of a square in the xy plane (as viewed along z). This behaviour of no correlation for $|n, n\rangle$ and correlation for $|n + 2, n\rangle_-$ repeats as n (i.e. B) increases with the $|n + 2, n\rangle_-$ distribution becoming increasingly point-like (i.e. classical). The states $|n + 2, n\rangle_-$ eventually dominate at high B; it is however remarkable that the classical configuration is *not* approached monatonically with increasing B field. Interestingly, the possible experimental observation of a bilayer Wigner crystal at $\nu = \frac{1}{2}$ has recently been reported in a wide quantum well [10].

(a)

(b)
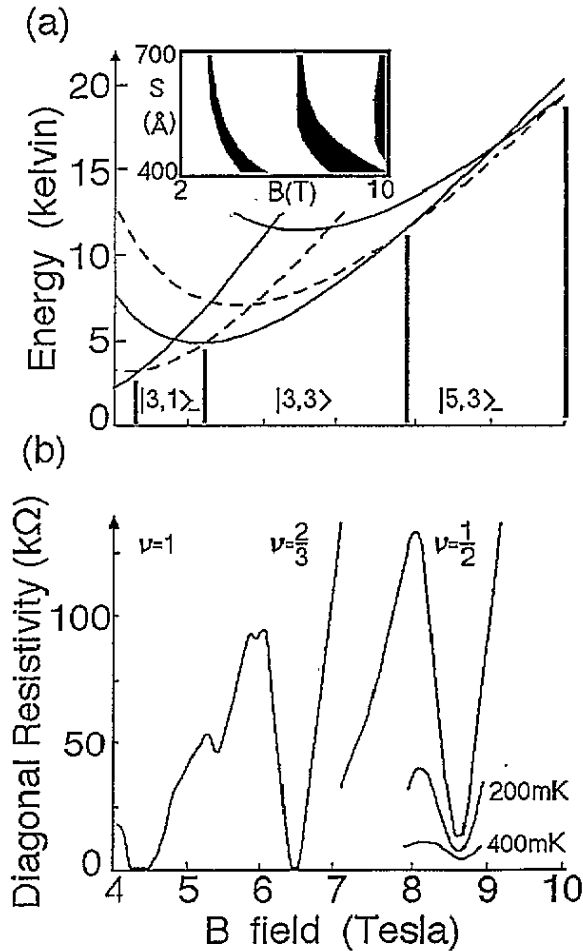
Figure 1. (a) Theoretical energies (measured in Kelvin relative to an arbitrary zero) for eigenstates $|n, n\rangle$ (solid lines) and $|n+2, n\rangle_-$ (dashed lines) as a function of B. Lowest curve at a given B gives ground state. Parameters given in text. Effective ν values for each ground state (see text) are identical to the experimental filling factors directly below in figure 1(b). Ground states $|1, 1\rangle$ ($\nu = 2$; $B \sim 2 - 3$ T) and $|5, 5\rangle$ ($\nu = \frac{2}{5}$; $B > 10$ T) are off-scale. Inset shows B-field ranges of stability of these states with increasing s. with increasing s. $\nu = 2, \frac{2}{3}$ and $\frac{2}{5}$ (white regions) remain stable. (b) Experimental results for diagonal resistivity at $T = 0.15$ K (from figure 1 of [1]). For clarity, Hall resistivity is not shown.

Figure 3 compares the charge densities of the analytically obtained $\nu = 1$ eigenstate $|3, 1\rangle_-$ and the Jastrow, or so-called Greek–Roman, (111) wavefunction[4]. Compared with the eigenstate $|3, 1\rangle_-$, the (111) form *over*estimates the inter-layer correlation since it essentially treats all four electrons as if they occupied the same layer; it also lacks any explicit s dependence. The fundamental distinctions between the two wavefunctions show up in their functional forms. The (unnormalized) wavefunction $|n + 2, n\rangle_-$ written in polar coordinates ($u_2 = (u_2, \phi_2)$ etc) becomes

$$u_2^{(\beta m^*/\hbar^2 + (n+2)^2)^{1/2}} u_3^{(\beta m^*/\hbar^2 + n^2)^{1/2}} e^{i(n+2)\phi_2} e^{in\phi_3}$$
$$-u_2^{(\beta m^*/\hbar^2 + n^2)^{1/2}} u_3^{(\beta m^*/\hbar^2 + (n+2)^2)^{1/2}} e^{in\phi_2} e^{i(n+2)\phi_3} \tag{5}$$
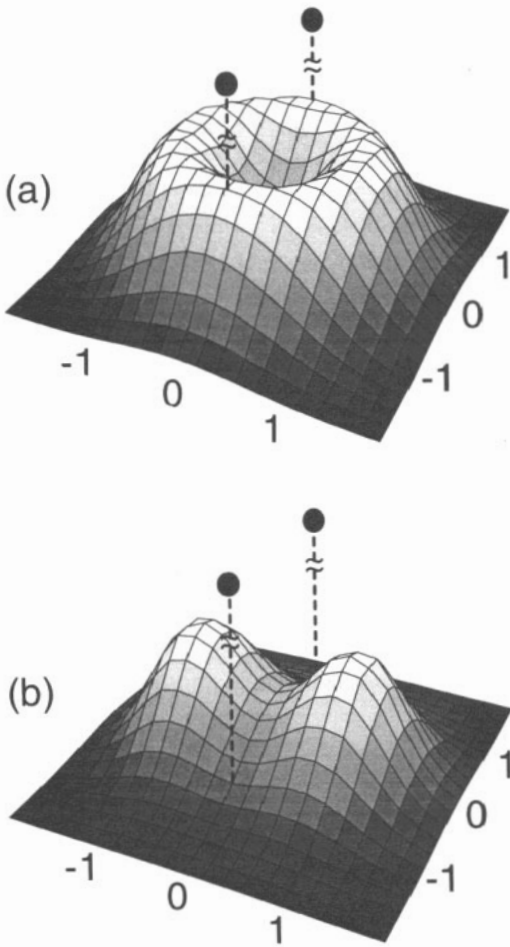
Figure 2. Charge density of electrons in a given layer for (a) eigenstate $|3, 3\rangle$ and (b) eigenstate $|3, 1\rangle_-$. Electrons in the other layer are fixed diametrically opposite each other at $x = \pm1$, $y = 0$ as indicated schematically. Parameters as for figure 1.
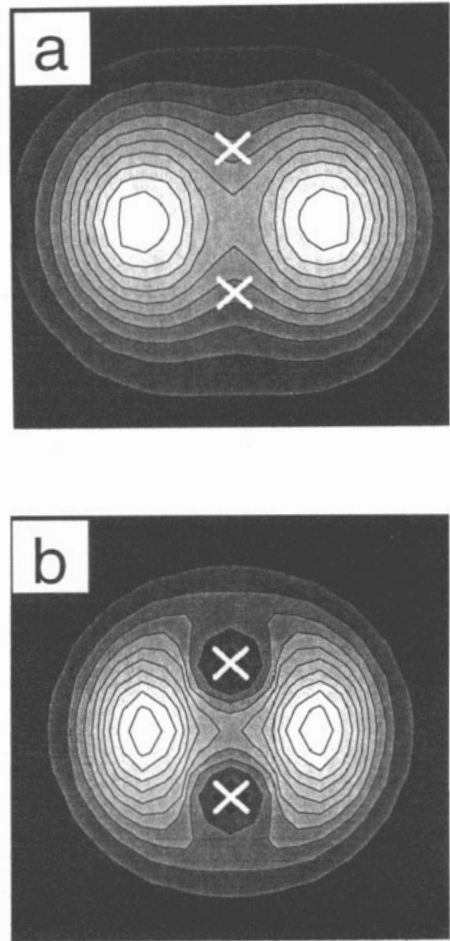
Figure 3. Charge density contours of electrons in a given layer for (a) analytically obtained $\nu = 1$ eigenstate $|3, 1\rangle_-$ and (b) Greek–Roman (111) wavefunction. Electrons in the other layer are fixed diametrically opposite each other (shown by $\times$) as in figure 2. Parameters as for figure 1.

multiplied by $\exp[-\frac{m^*}{2\hbar}(\omega_1(B)u_1^2 + \omega_2(B)u_2^2 + \omega_3(B)u_3^2 + \omega_4(B)u_4^2)]$. Note that $\omega_2(B)$, $\omega_3(B)$ and $\omega_4(B)$ are s dependent. The powers of the u in (5) are irrational and the wavefunction includes mixing with all Landau levels (i.e. contains z and z^* [11]). This is in contrast to the (Jastrow) Greek–Roman functions which are built from the lowest Landau level (i.e. functions of z [4]). To compare directly with the Greek–Roman form, we define $z_i = |r_{i,1}|e^{i\phi_{i,1}}$ and $z_{[i]} = |r_{i,2}|e^{i\phi_{i,2}}$. Since the Greek–Roman form is actually an eigenstate of the *non*-interacting double-layer system, we must also take the limit of vanishingly small interaction ($\beta \to 0$). In this limit, (5) formally becomes

$$\prod_{1\leqslant i<j\leqslant 2}(z_i - z_j)^{n'} \prod_{1\leqslant i<j\leqslant 2}(z_{[i]} - z_{[j]})^{n} - \prod_{1\leqslant i<j\leqslant 2}(z_i - z_j)^{n} \prod_{1\leqslant i<j\leqslant 2}(z_{[i]} - z_{[j]})^{n'} \qquad (6)$$

with $n' = n + 2$. The (mmn) Greek–Roman form is

$$\prod_{1\leqslant i<j\leqslant 2} (z_i - z_j)^m \prod_{1\leqslant i<j\leqslant 2} (z_{[i]} - z_{[j]})^m \prod_{1\leqslant i,j\leqslant 2} (z_i - z_{[j]})^n \tag{7}$$

and is fundamentally different from the entangled form of (6) and hence (5).

We finish by emphasizing that although our study only considers small N, the present results do represent the first analytically obtained solutions to a double-layer electron problem containing both intra- and inter-layer interactions.

References

[1] Eisenstein J P, Boebinger G S, Pfeiffer L N, West K W and He S 1992 *Phys. Rev. Lett.* **68** 1383
[2] Suen Y W, Engel L W, Santos M B, Shayegan M and Tsui D C 1992 *Phys. Rev. Lett.* **68** 1379
   Murphy S Q, Eisenstein J P, Boebinger G S, Pfeiffer L N and West K W 1995 *Phys. Rev. Lett.* **72** 728
[3] Laughlin R B 1983 *Phys. Rev. Lett.* **50** 1395
   Halperin B I 1983 *Helv. Phys. Acta* **56** 75
   Jain J K 1993 *Adv. Phys.* **41** 105
[4] He S, Xie X C, Das Sarma S and Zhang F C 1991 *Phys. Rev.* B **43** 9339
   He S, Das Sarma S and Xie X C 1993 *Phys. Rev.* B **47** 4394
   Yoshioka D, MacDonald A H and Girvin S M 1989 *Phys. Rev.* B **39** 1932
[5] Cote R, Brey L and MacDonald A H 1992 *Phys. Rev.* B **46** 10239
   Yang K, Moon K, Zheng L, MacDonald A H, Girvin S M, Yoshioka D and Zhang S-C 1994 *Phys. Rev. Lett.* **72** 732
   Chakraborty T and Pietilainen P 1987 *Phys. Rev. Lett.* **59** 2784
   Rezayi E H and Haldane F D M 1987 *Bull. Am. Phys. Soc.* **32** 892
   Fertig H A 1989 *Phys. Rev.* B **40** 1087
   Brey L 1990 *Phys. Rev. Lett.* **65** 903
[6] Laughlin R B 1983 *Phys. Rev.* B **27** 3383
[7] Quiroga L, Ardila D R and Johnson N F 1993 *Solid State Commun.* **86** 775
[8] Kasner M and Apel W 1994 *Trieste Preprint No 9405076*
   For $N = 2$ this definition coincides with the form $\nu = N(N - 1)/2J$ quoted by Maksym P A and Chakraborty T 1990 *Phys. Rev. Lett.* **65** 108
[9] Benjamin S and Johnson N F unpublished
[10] Suen Y W, Manoharan H C, Ying X, Santos M B and Shayegan M 1994 *Phys. Rev. Lett.* **72** 3405
[11] Johnson N F and Quiroga L 1994 *Solid State Commun.* **89** 661